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CA1 2.1 Tangents Lab

Purpose: This activity will teach you what the tangent function means and how to use it on your calculator. It will be used in several activities throughout the year.

Materials needed: ruler, protractor, scientific calculator with tangent key.

Procedure:

1. When studying triangles, there's usually an angle you know or an angle you want to know. Let's call this the *angle of interest* and call it by the Greek letter *theta* (θ). Measure theta in Figure 1 and write down the angle you get here. The right angle is always opposite to the hypotenuse, and the hypotenuse is always the longest side.

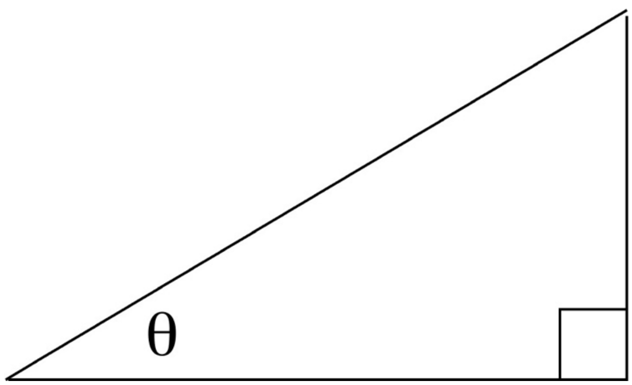

Figure 1. A right triangle has one 90 degree angle.

2. Label the drawing as follows: The longest leg of the triangle is called the *hypotenuse* (h). The bottom of the triangle is called the *adjacent* leg, and the vertical line is called the *opposite* leg.

3. What are these legs adjacent to and opposite from?

4. Measure the length of the opposite leg to the nearest 0.1 cm and record the value here: ____________

5. Measure the length of the adjacent leg to the nearest 0.1 cm and record the value here: ____________

6. Divide the opposite side (answer #4) by the adjacent leg (answer #5) and write your answer here. ____________

CA1 2.1 Tangents Lab

7. Here is another triangle (Figure 2) identical to the first one, except it is somewhat larger. Measure the same legs in the same way and again calculate the answer.

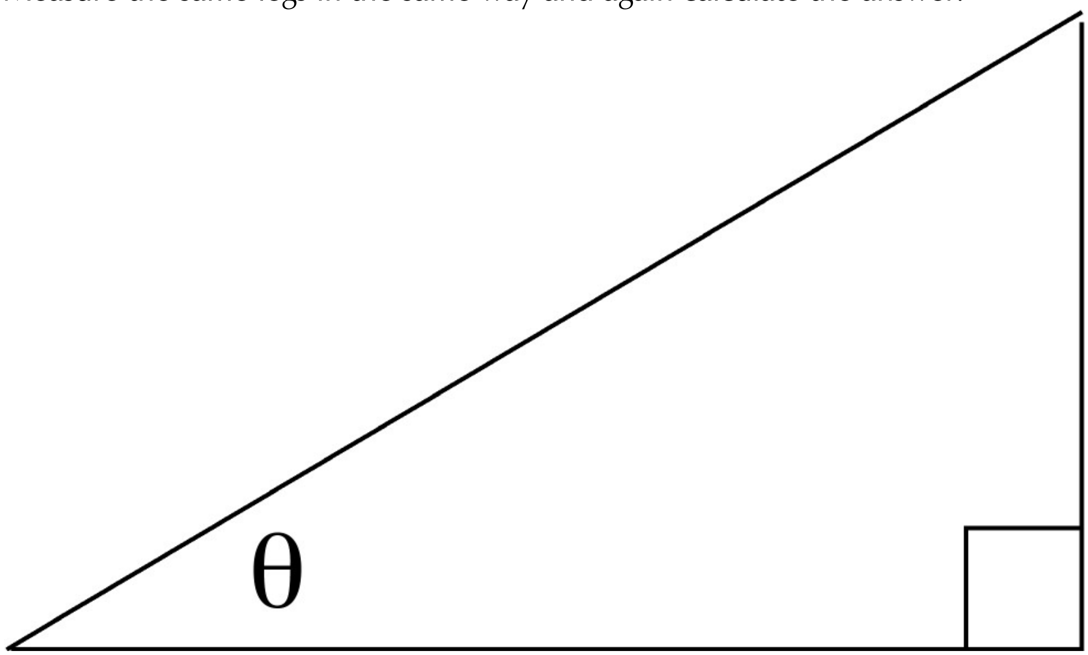

Figure 2. Another triangle, proportional to the first.

opposite leg:__________

adjacent leg: __________

ratio of opposite to adjacent: __________

8. How do the two answers compare?

9. Would this work if the triangle were exactly the same shape, but larger—say the size of a flagpole?

10. Would it work if the distance from the left bottom corner to the upper right corner were the distance from the earth to the sun?

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CA1 2.1 Tangents Lab

11. It turns out that for every value of theta, there is a unique number which is identical for any triangle with that specific shape. Different triangles have different values. In the space below, draw another right triangle with a different shape than the ones on this page. Designate one angle as "theta," then determine the ratio of the opposite side's length and the adjacent side's length as before.

Record your answers here:

Angle____

The ratio of the opposite side to the adjacent side of a triangle is called the *tangent* of the angle. It is usually written like this:

tan (θ) = a number

12. Wouldn't it be great if there were a table of numbers which correspond to the answers you would get if you did this for a variety of triangles? Such tables used to be published regularly in math books. In fact, whole books were published with nothing but lists of angles and their tangents, square roots, and other functions.

If you know the tangent of an angle, you can figure out what the angle is as well. This is called taking the arc tan or inverse tangent or $\tan^{-1}$ of a number. It is usually activated using a 2^{nd} function or inverse key on your calculator.

What the inverse tangent of 0.60? ____

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CA1 2.1 Tangents Lab

This page intentionally left blank; however, your teacher may ask you to draw additional triangles to analyze in this space.

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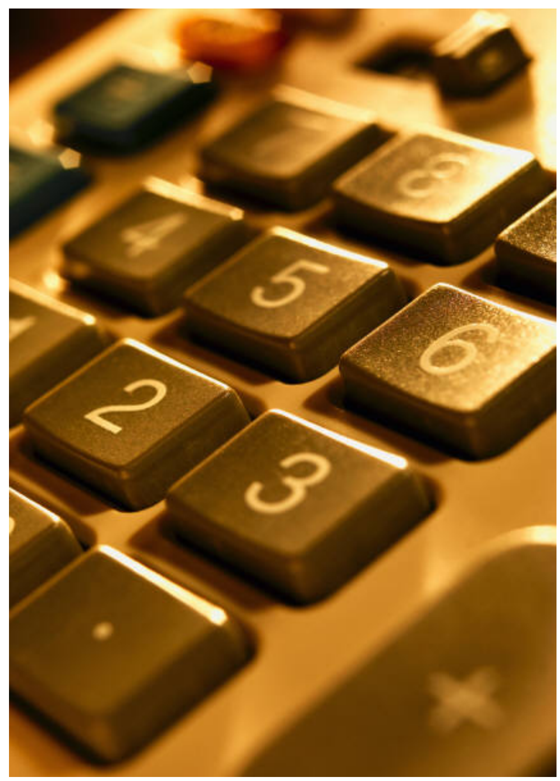
CA1 2.2: Tangent Practice Handout

Can you imagine having a whole book with nothing but numbers, used to look up tangents and other trig functions? Books like this used to be on every scientist's bookshelf (and still are, if they are old enough and sentimental about it like your author.)

Luckily for you, a scientific calculator has the equivalent of a tangent table hundreds of pages long built in. All you have to do (depending on the brand of your calculator) is enter an angle and press *tan*. Using a scientific calculator, find the tangent of the angles from the table below.

If you know a tangent but don't know the angle (for example, 0.47 in the table above) then enter the tangent and use the *inverse tan* or *2nd fn tan* or *arctan* function to reverse the lookup process. This might also be labeled $\tan^{-1}$ on your calculator. You need to press the shift or 2nd function key first, and then tan, usually.

Angle	**tan (angle)**
0	0.00
5	0.09
10	
15	0.27
	0.36
	0.47
30	
35	
40	
45	
50	
	1.43
	1.73
	2.14
	2.75
	3.73
	5.67
85	
90	undefined



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CA1 2.20: Scientific Notation Handout

Purpose: to learn how to enter scientific notation in your calculator and perform a few basic calculations.

Scientists and astronomers use scientific notation to avoid writing a large number of place-value zeros. For example, when we write "one billion,"
we could write

1,000,000,000

Or we could write "1 followed by 9 zeroes," which saves us no effort.

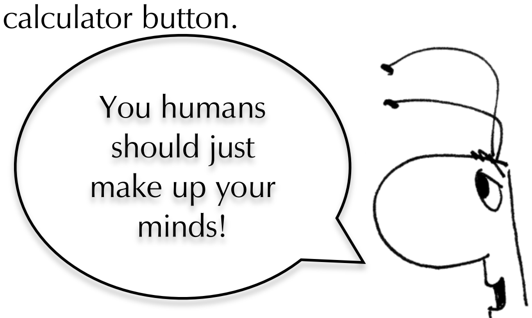
In scientific notation, we would write "1×10^9" which is read "One times ten to the ninth power" or more typically "1 times ten to the ninth," which means move the decimal place nine places to the right, adding nine zeroes.

On a calculator, there are a limited number of digits that can be displayed. Try multiplying 1 billion by 1 billion and you will see the 18 zeroes required will not fit.

So, calculator manufacturers have devised a number of ways to show these large numbers.

Depending on the calculator, you might try pressing one of the following buttons. In the chart below symbols in square brackets [ ] represent a calculator button.

To enter 1×10^9, try these key combinations:



Calculator brand	keys
Casio	[1] [EXP] [9]
TI	[1] [^] [9]
HP	[1] [EE] [9]
graphing calculators	[1] [10^x] [9]
varies	[1] [2nd] [10^x] [9]
varies	[1] [$x10^x$] [9]

Other combinations are possible. To enter a negative exponent, you may need to press the [+/-] key *after* you enter the exponent, or in some cases *before*. Some calculators label the "change sign" key with an extra minus button on the keypad [(-)]. If all else fails, try the manual.

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CA1 2.20: Scientific Notation Handout

1. Express the following without scientific notation.

a. 9×10^{9}

b. 3.15×10^{6}

c. 150×10^{6}

d. 2.897×10^{-6}

e. -32.5×10^{-3}

2. Express the following in scientific notation:

a. 93,000,000

b. 6,000,000,000

c. 0.02394

d. -45000

e. -23.03×10^{-5}

3. Add the following and express the result in scientific notation:

a. $(45 \times 10^{4}) + (60 \times 10^{5})$

b. $(12 \times 10^{2}) + (45 \times 10^{2})$

c. $(-56 \times 10^{9}) + (103 \times 10^{8})$

4. Multiply the following numbers. Express the result in scientific notation.

a. $(10 \times 10^{10}) \bullet (20 \times 10^{10})$

b. $(6.59 \times 10^{-34}) \bullet (106 \times 10^{40})$

c. $(123 \times 10^{6}) \bullet (-4.5 \times 10^{-4})$

Why do astronomers and scientists use scientific notation? Isn't it easier just to write out the number?

____

____

____

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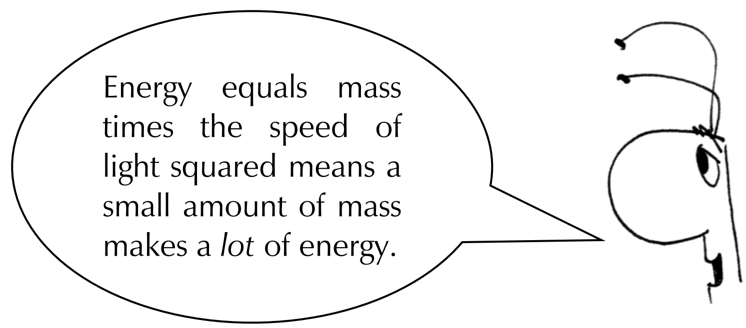
CA1 2.21: Age of the Sun Handout

Purpose: To use estimations and ratios to determine the age of the sun and how long it will last until it dies.

Background: The sun gets its energy from nuclear fusion. In a nuclear fusion reaction, four hydrogen atoms combine to form a helium atom. In the process some mass is lost, and this mass is converted to energy according to Einstein's equation $E = mc^2$. Using some basic facts about the sun, we can estimate the rate of burning and the available fuel to determine how long the sun will last.

The solar constant is the energy received by the earth, per square meter, from the sun. The total energy output is equal to the energy that hits a sphere the size of the earth's orbit. So, we take the solar constant times the surface area of a sphere with the radius of the earth's orbit.



Each step leads to the next answer. Be careful because one wrong answer will cascade throughout the worksheet to a final answer that is incorrect.

To help you keep track of the steps, each answer is referred to by line number in subsequent calculations. Certain answers are given partially as check points to help you avoid mistakes.

Constants

1370	Solar Constant (W/m^2)
$1.5x10^{11}$	radius of the earth's orbit (m)
$1.99x10^{30}$	mass of the sun (kg)
$1.67325x10^{-27}$	mass of one hydrogen atom (kg)
$6.645x10^{-27}$	mass of one helium atom (kg)
300,000,000	speed of light (m/s)

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CA1 2.21: The Age of the Sun

Part One. Determine the energy output of the entire sun.	
1. Find the surface area of a sphere that has a radius equal to the radius of the earth's orbit by using the formula $A = 4\pi r^2$. Use the earth's orbital radius for *r* . (Find it on the previous page.)	
2. Multiply the answer to line 1 by the solar constant in Watts per square meter to find the total energy output of the sun. Note: a Watt is a Joule / sec, or energy per unit time.	
Part Two. Energy Output from Hydrogen Fusion. The sun is powered by hydrogen fusion. In hydrogen fusion, 4 hydrogen atoms combine to form one helium atom.	
3. Find the mass of four hydrogen atoms.	
4. Subtract the mass one helium atom.	
5. Find out how much energy this conversion generates by taking the difference (Answer to Line 4) and multiplying it by the speed of light squared. This is the application of $E = mc^2$. The answer comes out in Joules.	
Part Three. Find the rate at which conversions occur in the sun.	
6. Since the sun's total energy output is known (Line 2) for one second, we can compute the rate of conversions per second by dividing the energy output per second (line 2) by the energy for each conversion (Line 5). The answer is in "conversions per second." $$\frac{\left(\frac{\text{Energy output of sun}}{\text{second}}\right)}{\left(\frac{\text{Energy provided by}}{\text{one conversion}}\right)} = \frac{\text{Line 2}}{\text{Line 5}} =$$	
Part Four. Estimate the amount of fuel (Hydrogen) available in the sun for conversion.	
7. Hydrogen fusion can only occur in the central, hot core of the sun, which contains about 10% of the mass of the entire sun. Divide the mass of the sun by 10. Answer is in kilograms. (HINT: The answer is very nearly 2.0 x 10 to the something…)	
8. Assume that when the sun was formed, like most of the rest of the universe, it was made of about 90% hydrogen and 10% other. Multiply Line 7 by 0.90.	
9. Find out how many hydrogen atoms this represents by taking the available fuel mass (line 8) and dividing by the mass of one hydrogen atom.	

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CA1 3.14: The Height of Lunar Mountains Lab

Purpose: To analyze a photo to determine the height of a lunar mountain.

Equipment: Ruler (cm), photo of Tycho, NIH Image software (optional).

Background: Many craters such as Tycho have a central peak that is surrounded by a relatively flat floor. Shadows of mountains cast on the floor can be used to determine the height of the mountain through an application of geometry and trigonometry.

Procedure: You will analyze a photo of a central peak in a lunar crater by measuring the length of its shadow and using geometry to determine the height of the mountain. The first thing you need to do is gather some information.

1. From Appendix A-3, find the diameter of the moon and divide it by two to determine the radius of the moon. This will be called *r* throughout this problem.

2. Review CA1 3.1 and CA1 3.3 to determine the general position of the moon during waning gibbous phase. Draw a sketch of the appearance of the moon from the earth and the position of the moon in its orbit as seen from far above the earth's North Pole.

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CA1 3.14: The Height of Lunar Mountains Lab

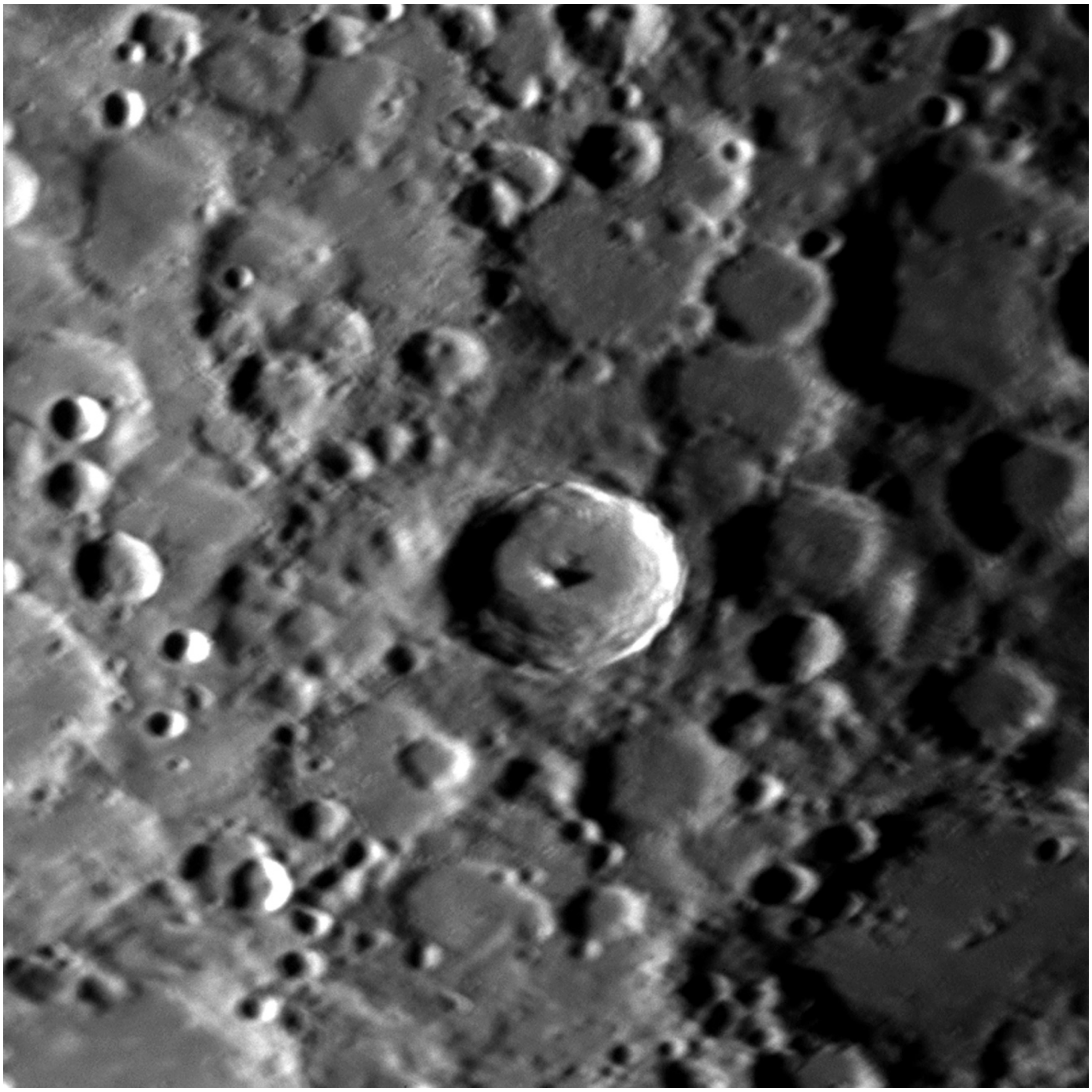

Figure 1. This photo of the crater Tycho was taken October 17, 2003 by Jim Scala, an experienced astrophotographer. The exact time of the photo was 13h 10m 47seconds Universal Time, which was about 5:10 AM on the West Coast, 8:10 AM on the East Coast.

3. From what direction is the light coming in this photo?

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CA1 3.14: The Height of Lunar Mountains Lab

4. Does this seem reasonable given that the crater is on the side of the moon facing the earth? (In other words, is the picture aligned correctly or reversed? Some telescopes reverse the images taken through them.)

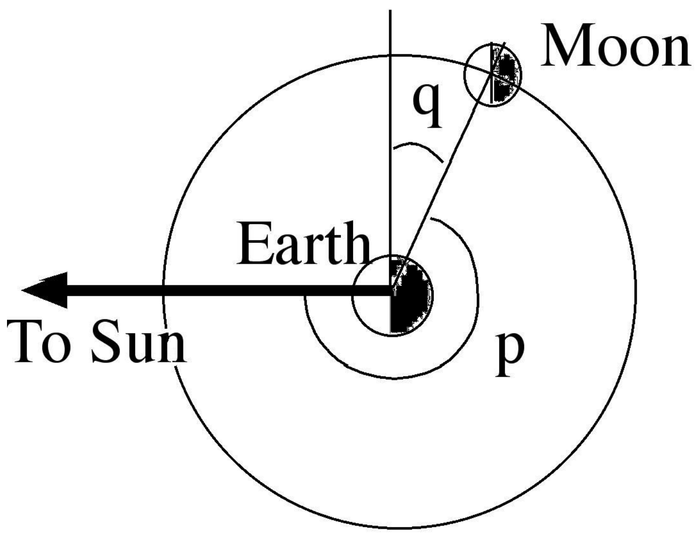

Figure 2. Geometry of the moon's waning gibbous phase at the time the photo was taken.

5. At the time of this photo, the moon was 21.2 days "old" which means 21.2 days had passed since the last new moon. The moon takes 29.5307 days to complete a cycle of phases, so this can be used to tell what fraction of the moon's orbit had been completed at the time the photo was taken.
The ratio of days is the same as the ratio of angle p to a complete circle.

$$\frac{\text{"age" of moon}}{29.5307 \text{ days}} = \frac{\text{angle p}}{360 \text{ degrees}}$$

Use this relationship to determine angle p.

6. Angle q is the angle between the Earth-Moon line and the position the moon would be at third quarter phase. This becomes important later in the calculation. You can get angle q by finding 270 degrees – angle p. Write down angle q here.

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CA1 3.14: The Height of Lunar Mountains Lab

The next step is to figure out the angle of the sunlight on the mountain, as seen from an observer on the moon.

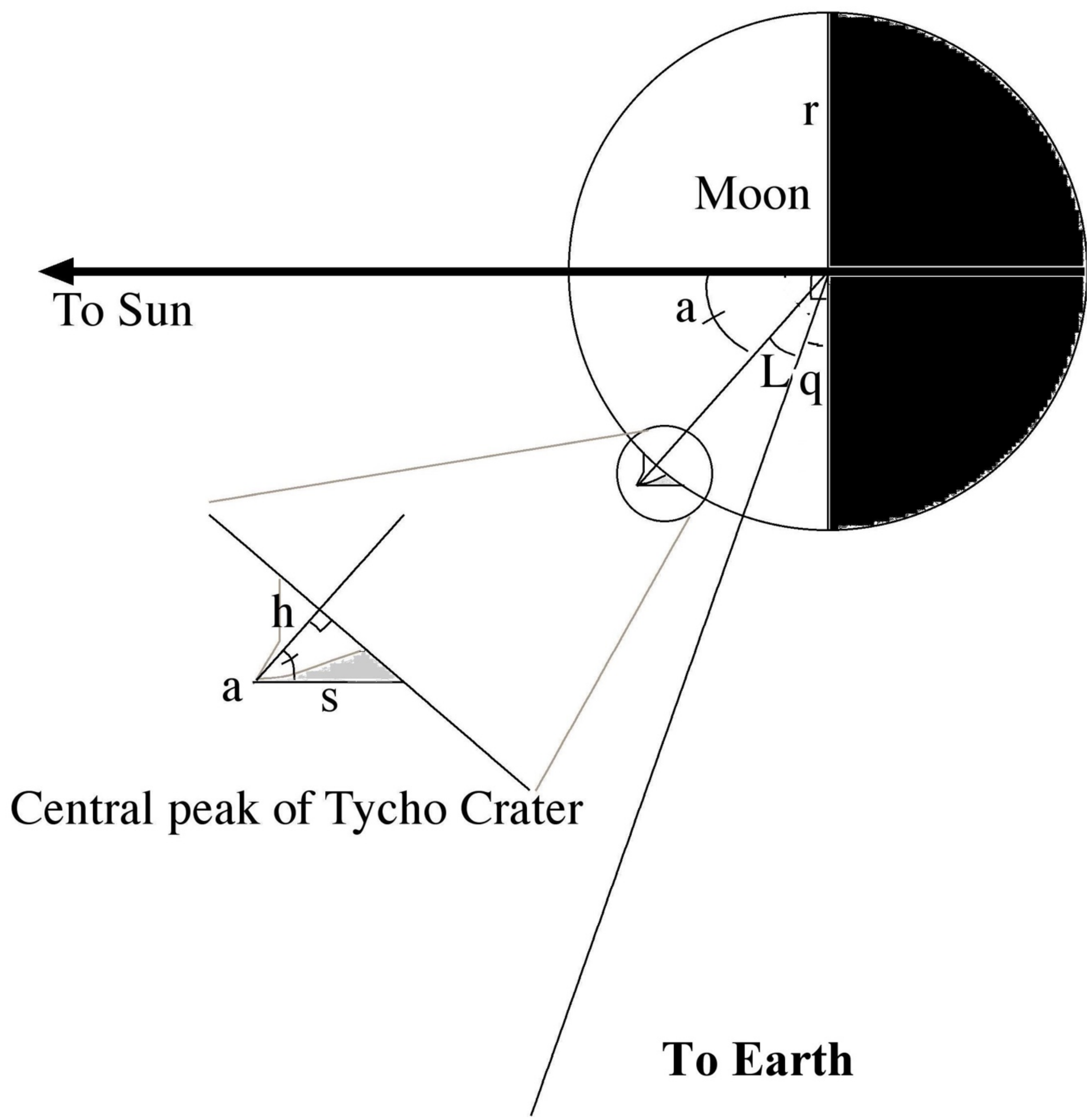

Figure 3: Geometry of the shadow of the central peak of Tycho Crater. In this diagram q is determined by the moon's phase, L is the longitude of the crater being investigated, r is the radius of the moon, s is the length of the shadow of the mountain, a is an angle derived from L and q, and h is the height of the mountain.

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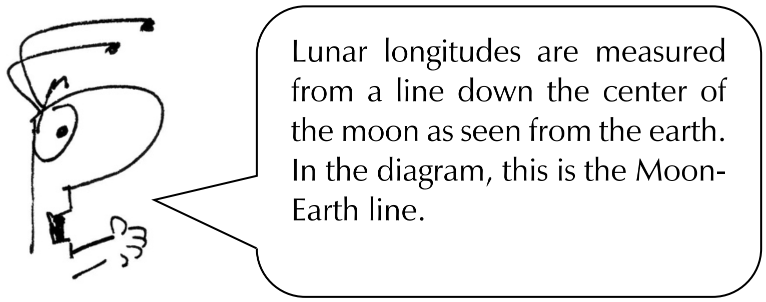
CA1 3.14: The Height of Lunar Mountains Lab



7. The next step is to determine angle a. Angle L is the longitude of Tycho crater, which can be looked up in a lunar ephemeris or map. The longitude of Tycho is 11 degrees West.

Compute the value of angle a, write it down here.

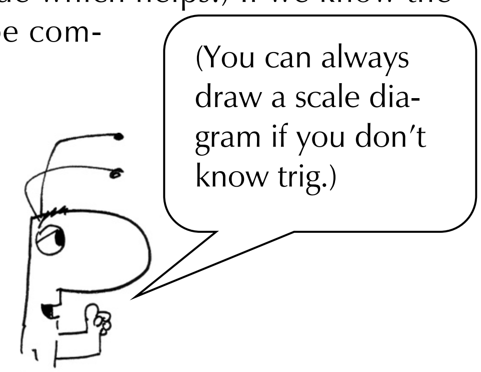
a = 90-q-L

8. Now look at the inset—this shows the angles around the mountain at the center of the crater. s is the length of the shadow, which must be determined in km if you wish to know the height of the mountain in km.

The length of the shadow s can be computed using a ratio that automatically takes into account foreshortening due to the latitude of the crater and the angle of sunlight on the moon's surface. (Tycho is near 0 degrees Lunar longitude which helps.) If we know the size of the crater, then the length of the shadow s can be computed using

$$\frac{\text{length of shadow in picture (cm)}}{\text{diameter of crater in picture (cm)}} = \frac{s}{\text{diameter of crater (km)}}$$

Tycho is approximately 85 km in diameter. Use this ratio to determine the length of s.



9. Again referring to the inset picture, Angle a and the shadow length are related to h by the trig relation

cosine a = h/s

Solve this equation for h, and compute the estimated height.

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CA1 3.14: The Height of Lunar Mountains Lab

10. Compare this result to the diameter of the mountain, and look at the picture again. Does your answer seem reasonable?

Challenge: Use your own lunar photograph of a moon crater and determine the height of the peak.

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CA1 3.15 What Have We Learned About the Moon?

Purpose: To learn about the Apollo Missions via a Web Quest. To learn what we found out about moon rocks and the origin of the moon. To understand how we went to the moon and why.

Equipment needed: Internet access or a general encyclopedia.

Procedure: The web quest is divided into four parts, listed below. Divide the sections up within the class (one section per row, perhaps) and assign each person one or two questions. During the time provided by the teacher for research, look up the answers to the questions via the web. Record the answer, the URL of the page where you found it, and evaluate the trustworthiness of the answer by noting whether or not the page has a named author, a date of publication, and a bibliography page. The answer to each question should also include what date the event occurred, if appropriate.

At the end of the research time, your teacher may ask you to make a presentation (poster, PowerPoint, or simply go over the answers) that tells the story of your assignment.

Sections:
The Space Race
How Apollo worked
What We Learned
Other Lunar Missions

The Space Race:
1. Who was the first man in space?
2. Why was there a "space race" in the early 1960's?
3. Who was the first person to "walk in space"?
4. Which country first sent an animal into space?
5. Which country first sent a woman into space?
6. What was the first manned American rocket capsule called?
7. How many astronauts were in that program?
8. Why was the Gemini program named after that particular constellation?
9. List two things the Gemini program accomplished.
10. Who was the first American to orbit the earth?

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CA1 6.11: Kepler's Laws Problems Handout

1. Write down each of Kepler's Laws here:
1st Law:

____

____

2nd Law:

____

____

3rd Law:

____

____

____

____

1. A planet orbits the sun in 100 years. How many AU is it from the sun?

____

____

____

____

2. A planet orbits the sun at a distance three times greater than the distance of the earth to the sun. How long is its year?

____

____

____

____

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CA1 6.11: Kepler's Laws Problems Handout

4. A comet has an orbital period of 200 years. Its orbital eccentricity is 0.9. How far from the sun is it when it is moving the fastest? the slowest?

____

____

____

____

5. What kinds of orbits have the greatest speed differentials? (This means the greatest difference between its fastest and slowest speeds.)

greatest

____

slowest

____

6. A new object named Quaoar was recently discovered. It is called a Trans-Neptunian Object (TNO). Its orbital period is 286 years. Why is it called a TNO?

____

____

7. Halley's comet takes 76 years to orbit the sun. Calculate the length of its semi-major axis. Given that the eccentricity of its orbit is 0.967, determine its perihelion and aphelion distances.

____

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CA1 6.14: The Size of Venus' Orbit Lab

Purpose: to measure the size of Venus' orbit based on observations of its motion in the sky.

Background: Venus is closer to the sun than the earth. Because of this, Venus will only appear so far away from the sun. It can't be seen rising in the east as the sun is setting in the west, because there is no place in Venus' orbit which places it behind us with respect to the sun.

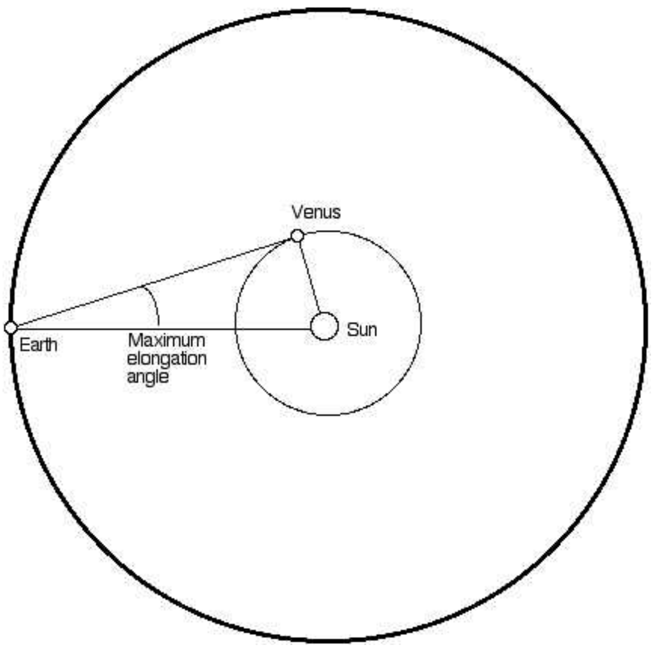

Figure 1. The maximum elongation of Venus. (Not to scale)

This diagram shows why Venus can never be seen in opposition to the earth.

Since Venus is always closer to the sun than the earth, there is a limit to the size of the angle between the sun and Venus. This *elongation angle* reaches a maximum when Venus is in the position shown. Venus' orbit is nearly circular, but Earth's is less so, and this is what causes the year-to-year variation.

Note that the angle between the Sun, Venus, and Earth (Venus as vertex) is 90 degrees, making this a right triangle.

For any right triangle, if you know a side and an angle, you can figure out the rest of the triangle.

Procedure: Here is a list of the sides and angles involved in this problem. Explain which two are easy to measure or look up and explain why the other ones are not easy to measure.

Getting the maximum elongation angle

If you are taking this class near a time of maximum elongation, your instructor may ask you to measure the elongation angle directly. However, odds are the timing isn't right, so you may have to resort to simulating the motion of Venus using a planetarium program such as Stellarium.

Here are the steps to obtain the maximum elongation angle using Stellarium.

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CA1 6.14: The Size of Venus' Orbit Lab

1. Start Stellarium. Turn off the ground and atmosphere. Click the Equatorial button (looks like a telescope).

2. "Find" the sun, and center on it.

3. Make sure the planet labels and orbits are on. It does not matter where you are located on the earth or the specific date.

4. Zoom out until you can see Venus.

5. Note the date you began the simulation on the simulator. Use the angle measuring tool (looks like an angle wedge) and draw a line between the sun and Venus. Record this angle, and the date, here.

____

____

____

6. Use the "jump ahead by 1 day button" and the "go back by 1 day button" move Venus back and forth in time. These are the square bracket keys [ ]. Note whether Venus gets farther or closer to the sun. Whatever day you choose to begin, adjust the date until Venus is as far from the sun as you can make it. At some point, it will reach a maximum. This is the maximum elongation angle. It is probably not necessary to adjust to the exact day, minute, and second the actual maximum occurs for this activity. Record the date and maximum elongation angle here.

____

____

____

7. Convert the degrees minutes and seconds recorded by the program as decimal degrees and write your answer below:

____

____

____

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CA1 6.14: The Size of Venus' Orbit Lab

8. Compare your results to the graph shown below. This graph shows the elongation angle of Venus as a function of time. Mark on the graph where the maximum elongation angle occurs and what its value is.

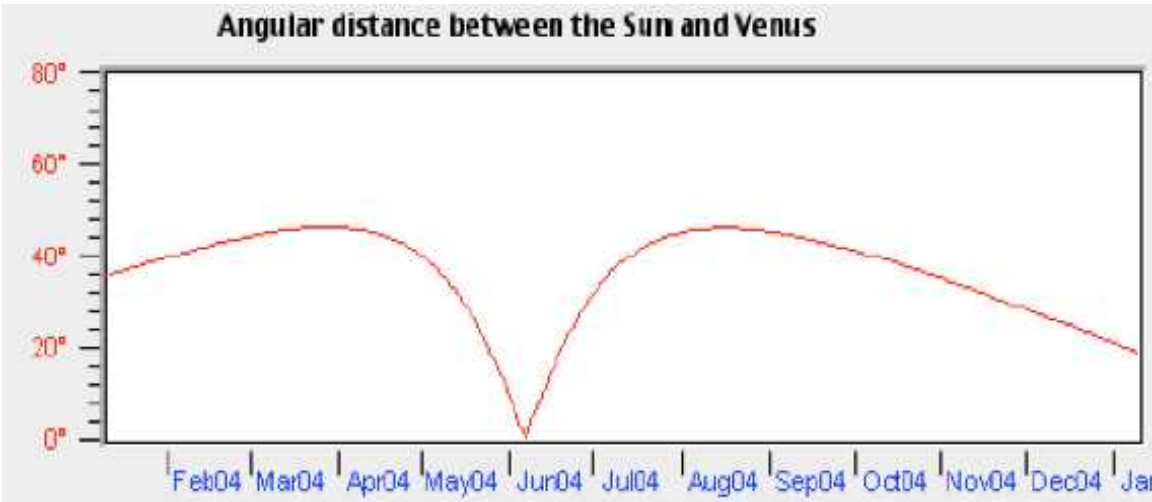

Figure 2. Elongation of Venus over time.

Finding the distance to Venus

You can use two methods here: a scale diagram using the information above, or a trig problem. Here are some hints:

Scale diagram method:	Trig method:
Draw the earth-sun distance as about half the width of your paper. Put the sun in the center of the paper. Figure out the scale of the drawing—how many centimeters is how many kilometers? When drawing the maximum elongation angle, remember the earth is the vertex, and the earth-Venus line is tangent to Venus' orbit.	The maximum elongation angle is the angle of interest. That means the Venus-Sun distance is the opposite side, and the Earth-Sun distance is the hypotenuse. Which trig function governs this solution?

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CA1 6.14: The Size of Venus' Orbit Lab

9. Whichever method you chose, show your calculations below. Circle your answer for the distance from Venus to the Sun, and be sure to indicate units.

____

____

____

Compare results

10. Compare your result for the orbital radius of Venus to the value published in a textbook. Calculate the relative error.

____

____

Questions:

1. For what other planet in the solar system will this method be effective for finding the distance of the planet to the sun?

____

2. Why won't this method work for superior planets?

____

____

3. Why can't Venus ever be in opposition to the earth?

____

____

4. If you looked at Venus through a telescope during maximum elongation, what would it look like?

____

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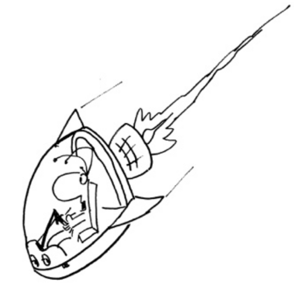
CA1 6.15: Design a Space Probe Mission Activity



Purpose: To design a one-way space probe mission to another planet.

Background: Most of what we know about the planets in detail, and all of what we know about many of the smaller moons in the solar system, has been discovered or measured by space probes. Surprisingly, most space probes are directed not at other planets, but at the earth itself (think of weather satellites, for example.) We can learn even more about the earth by studying other planets, because lessons we learn about planets in general can be applied to the earth in particular.

It is difficult to get from the earth directly to another planet, because our technology does not allow continuous thrust; the space probes must coast most of the way. While it is coasting, it is *orbiting* the sun, and is therefore controlled by Newton's and Kepler's Laws. We can use these laws to plan the trajectory of a space probe and determine how long it takes to get from one planet to another.

Transfer Orbits:

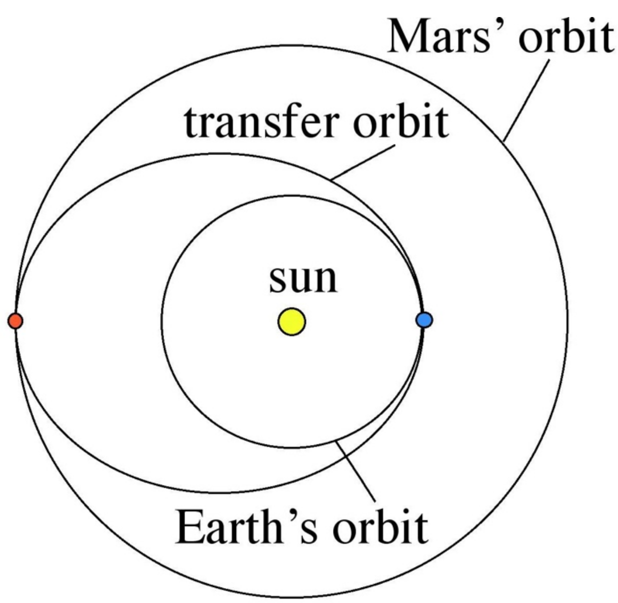


Figure 1. The transfer orbit begins at Earth and ends at Mars.

The most economical trip directly to another planet is called a "Hohmann Transfer Orbit" or a "Least Energy Transfer Orbit." Essentially, it starts from one planet, orbits the sun, and arrives at the other planet, 180 degrees from where it started. A transfer orbit to Mars is shown as an example.

The first step in planning the mission is to determine the characteristics of the transfer orbit. As can be seen in the diagram, the transfer orbit is an ellipse with a perihelion at the Earth, an aphelion at the target planet, and of course the sun at one focus. (If the transfer is to an inferior planet, the Earth is the aphelion.)

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CA1 6.15: Design a Space Probe Mission

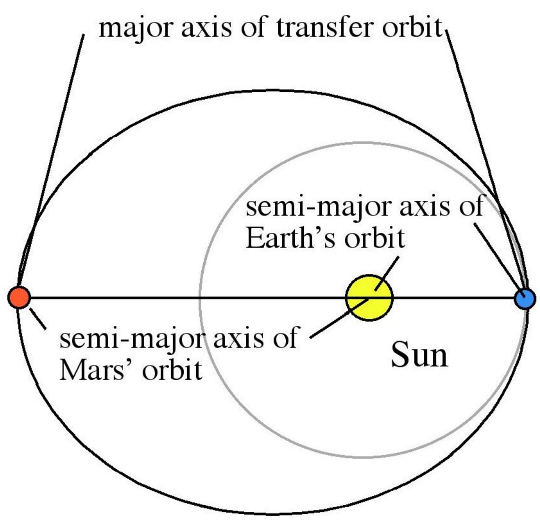


Figure 2. The major axis of the transfer orbit is equal to the sum of the semi-major axes of both planets involved in the transfer.

This means that the major axis of the transfer orbit is equal to the semi-major axis of Earth's orbit plus the semi-major axis of Mars' orbit.

Look up the size of the planet's orbit in Appendix A-2. Use the values in the AU column.

1. Semi-major axis of Earth's orbit:

____

2. Semi-major axis of Mars' orbit:

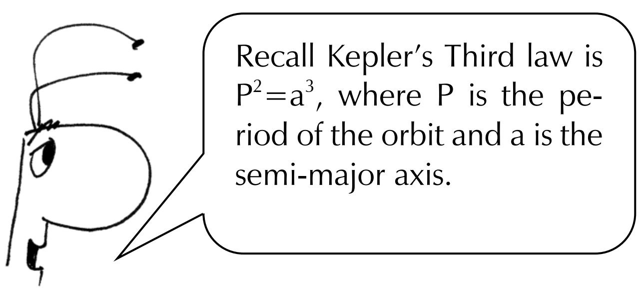
____

3. Now add these together to get the major axis of the transfer orbit.

____

4. What is the semi-major axis of the transfer orbit?

____



To plan this mission, we need to know how long the trip will take. The time it takes to orbit the sun is called the period of the orbit, and Kepler's Third Law will be applied to this orbit to find out.

5. Use the answer you got for the semi-major axis of the transfer orbit to calculate the period of the orbit. Show your work here.

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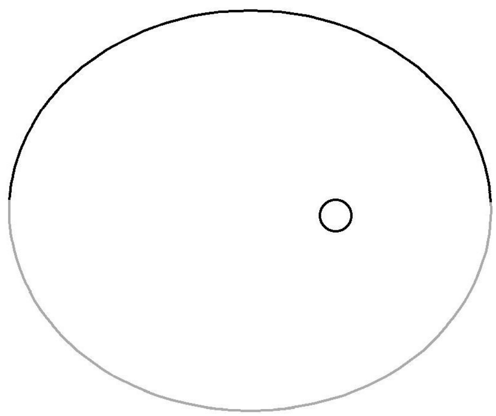
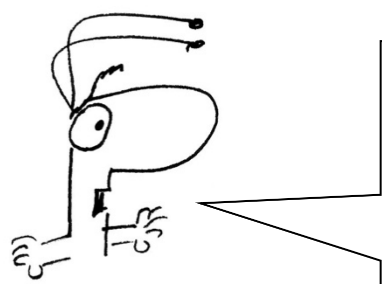


Figure 3. Only half of the orbital period is needed to complete a one-way trip.

For most space probes, we are not going to bring the probe back (and even if we completed the transfer orbit, the Earth would no longer be in the position it was when the probe left.)

6. Therefore we need to divide the orbital period by 2 to compute the *one way trip time*. **Put your answer in the box below** for how much time it would take for a "one way trip".

This is the time it would take to get to the planet! You've just completed the most important step in planning your space mission.



Unfortunately you can't just launch at any time you feel like it. If you leave at the wrong time, the target planet will not be 180 degrees from the departure planet when you arrive, and you'll be *lost in space forever*. Most spacecraft do not have enough fuel to make more than minor course corrections.

That means timing is critical. How do you know *when* to launch? The easiest way is to use a simulation. You need to find a time when the target planet is going to be 180 degrees from the launch planet after the time for the one way trip has passed. That sounds a bit confusing, so let's break it into steps.

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Step by Step instructions for determining launch dates

1. Find the one-way trip time. (Already done for our example.)

2. Figure out how many degrees along its orbit the target planet will travel in this time. To do this, divide the one-way trip time by the target planet's orbital period and multiply by 360.

$$\frac{\text{one-way trip time}}{\text{planet's orbital period}} x360 =$$

3. Subtract this angle from 180. The answer is the *leading angle*. On the diagram below, indicate where the leading angle is shown. Calculate the number and show the answer here.

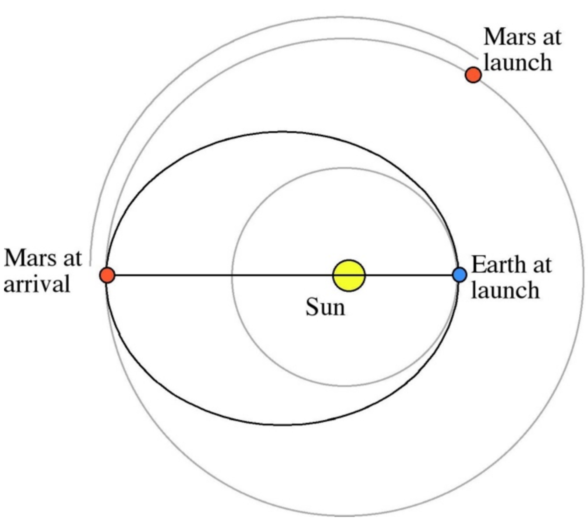

Figure 13. The position of Mars at launch has to be timed so Mars will be opposite the starting position by the time the space probe arrives at Mars.

4. [Optional] Using Celestia (www.shatters.net/Celestia) , find when the target planet is at the leading angle ahead of the Earth. Use a view above the sun and activate the planet's orbits. For more precision, compare the planets' ecliptic longitudes. When the target planet's ecliptic longitude is the leading angle larger than the earth's ecliptic longitude, it is time to launch. That is when you launch!

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CA1 6.15: Design a Space Probe Mission

Many space probes use transfer orbits like these. Others gain even more efficiency (at the expense of time) by using a slingshot effect around other planets. The most famous example is when the Voyager 2 space probe of the 1980's traveled to Jupiter, Saturn, Uranus and Neptune by using each planet's gravity to accelerate to the next.

Now it is time for you to determine your own mission to another planet. Pick a departure planet (it doesn't have to be earth) and a destination planet and repeat the steps on the previous page. Show all work and attach more paper if necessary. Use the recording space below for your answers.

Note: These are the *same steps* as on the previous example without the instructions embedded within them.

Starting planet: Semi major axis of orbit:

Destination planet: Semi major axis of orbit:

Major axis of transfer orbit:

Semi-major axis of transfer orbit:

Orbital period of transfer orbit:

Time for one-way trip:

Angle destination planet will move:

Leading angle for planet:
Caution: In many cases, the destination planet will have traveled more than one orbit before the space probe arrives. In this case, the leading angle may turn out to be greater than 180 degrees, or the answer could turn out negative. This step requires careful thought depending on the combination of planets selected.

Date when planets are configured correctly for launch:

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CA1 6.15: Design a Space Probe Mission

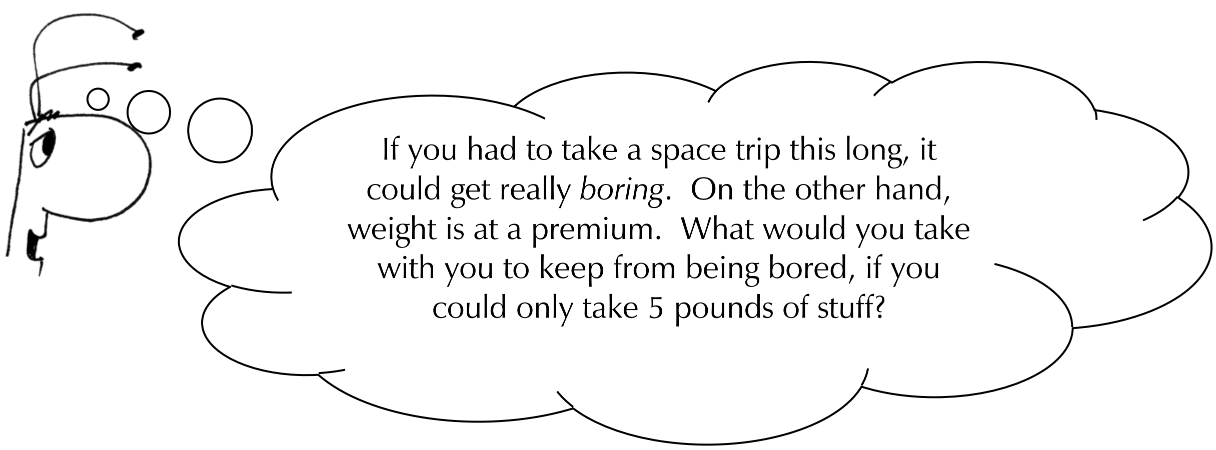
Questions:

1. To which planet will a transfer orbit take the longest time? _________________________

2. The shortest time? _________________________________

3. The Spirit and Opportunity space probes to Mars launched in June and July of 2003. When did they arrive? Compare this time to the length of time mentioned in the movie "The Martian" to make a supply ship go from earth to Mars to help a stranded astronaut.

____



Challenges:

1. Figure out the reverse trip for Mars: what is the configuration for a return trip to Earth? If you can match an Earth-to-Mars flight date with a Mars=to-Earth return flight date, how long would an astronaut visiting Mars have to stay?

2. Research famous space probes of the past (such as Viking) and show that the time between the launch date and arrival date is approximately equal to the one way trip time you computed. (Note: some variation is normal because the orbits aren't perfect circles and the rockets can accommodate launching at less than perfect times. The range of dates a rocket can launch and still meet its target is called the *launch window*.)

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CA1 6.16: Newton's Form of Kepler's Third Law Handout

Purpose: To show how basic physics can be used to derive Kepler's Third Law of Planetary Motion

Background:
There are only a few basic facts you will need to understand in order to see the physical basis for Kepler's Third Law of Planetary Motion.

> What we have to work with are the following facts:
>
> Centripetal Force: The force required to move any object in a circular path is $F_c = \frac{mv^2}{r}$
>
> Gravitational Force: The force exerted between any two objects in the universe $F_g = \frac{Gm_1m_2}{r^2}$
>
> Velocity of an object moving in a circle: $v = \frac{circumference}{period} = \frac{2\pi r}{P}$

When Isaac Newton first described the orbital motion of comets, he worked out the relationship between gravity and orbits (which even Kepler didn't understand) that explained why gravity caused Kepler's Laws to be true. This derivation is a simplified version for circular orbits. A similar derivation also explains elliptical orbits.

First, when planets move around the sun, they do so because the sun's gravity pulls on them. This gravitational force is given by the equation $F_g = \frac{Gm_1m_2}{r^2}$. In this equation, G is a constant, m_1 is the mass of the sun and m_2 is the mass of the planet. The variable r represents the orbital radius of a circular orbit.

Any object moving in a circular orbit, including a ball on a string, a car on a racetrack and a planet in an orbit, is subject to a force pulling it inward along a circular path. Such inward pulling forces are called Centripetal Force and is represented by the equation $F_c = \frac{mv^2}{r}$. In this equation, the m is the mass of the thing moving in a circle—in this case the planet. The variable v is the velocity of the planet, and r is again the radius of the orbit.

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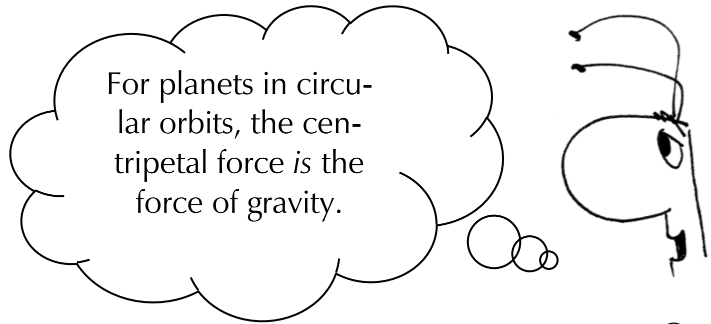
CA1 6.16: Newton's Form of Kepler's Third Law Handout

As we all know, in math class *is* means *equals,* so



$$F_g = F_c$$

$$\frac{Gm_1m_2}{r^2} = \frac{mv^2}{r}$$

Now m_2 on the left side is the planet, and m on the right side is the planet, so these variables cancel out. One of the r's from the left cancels the only one on the right, so now you are left with this:

$$\frac{Gm_1}{r} = \frac{v^2}{1}$$

Now we note that for any object moving in a circle, the speed is just the distance divided by the time, or

$$v = \frac{circumference}{period} = \frac{2\pi r}{P}$$

In this formula the term "period" means "time needed to go around the circle once."

Since this is v, v^2 is found by squaring the expression above.

$$v^2 = \frac{4\pi^2 r^2}{P^2}$$

The reason we want v^2 is that it already exists in the previous solution.

$$\frac{Gm_1}{r} = \frac{v^2}{1}$$

Since both these things are equal to v^2, they are equal to each other.

$$\frac{Gm_1}{r} = \frac{4\pi^2 r^2}{P^2}$$

or

$$P^2 = \frac{4\pi^2}{Gm_1} r^3$$

This last equation is known as *Newton's form of Kepler's Third Law (for circular orbits.)*

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CA1 6.16: Newton's Form of Kepler's Third Law Handout

Its usefulness can be seen in that the variable m is included. Given the period and radius of the orbit of any object, the mass of any parent body can be found. For example, the orbit of the earth can be used to "weigh" the sun; the orbit of Io can be used to find the mass of Jupiter, and so on.

If you realize that the term $\frac{4\pi^2}{Gm_1}$ is all constants, this law can be expressed as

$$P^2 \propto a^3$$

Which is identical to Kepler's Third Law, in which we use AU's and Earth years instead of metric units.

Use the space below to rewrite the algebra yourself, so you can see how it is done.

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CA1 6.17: The Mass of Jupiter Lab

Background: This classic lab activity is based on the observations of Jupiter's moons you made for CA1 5.10, Observing Jupiter's Moons.

Having made enough observations to determine the orbital period of one of the moons, you must now take your observations and convert them into metric units for use in a formula that will determine the mass of Jupiter. The formula is derived from Kepler's Laws, and analyzing it will tell us what information we need from the previous lab.

Kepler's third law of planetary motion is usually written by astronomers as

$$p^2 = a^3$$

but it is based on a formula derived from the laws of gravity and the definition of centripetal force (see Handout 6-3).

$$P^2 = \frac{4\pi^2}{Gm_1} r^3$$

. In this formula, r is the radius of a circular orbit and is equivalent to a. M is the

$$m = \frac{4\pi^2}{GP^2} r^3$$

mass of the thing being orbited, and the rest of the variables are constants. The Universal Gravitational Constant $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$. From this handout we can see that we need basically two pieces of information from our observations of Jupiter's moons: for a single moon, we need to know its *orbital period* and its *average distance from Jupiter*, both in metric units. This will allow us to calculate m, the mass of Jupiter.

Finding the orbital period

To get the orbital period, the easiest way is to select a moon for which you have collected data for more than a complete orbital period and measure the time it takes for the moon to return to its original position. If you do not have any moons for which you have a complete orbital period, it may be possible to estimate the period given just a fragment of the orbit.

Your teacher will advise you depending on the amount and quality of your data.

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CA1 6.17: The Mass of Jupiter Lab

Record the orbital period of the moon you selected here, and explain your answer.

1. Name of the moon: ____

2. Orbital period and explanation:

____

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Radius of the moon's orbit

Next we need to know the radius of this moon's orbit. You should have plotted the moon's position in Jupiter-diameters. The radius of the orbit is equal to the amplitude of your curve, or the maximum separation between the moon and Jupiter.

3. Record your maximum orbital separation here. This is the distance from Jupiter to the edge of your curve.

4. Look up the diameter of Jupiter and record it here. The *radius* of Jupiter is in the Appendix under "Planetary Physical Characteristics."

____

5. Convert your maximum separations (which are equal to semi-major axes of circular orbits) from "Jupiter diameters" into meters, using the information above. Show your work.

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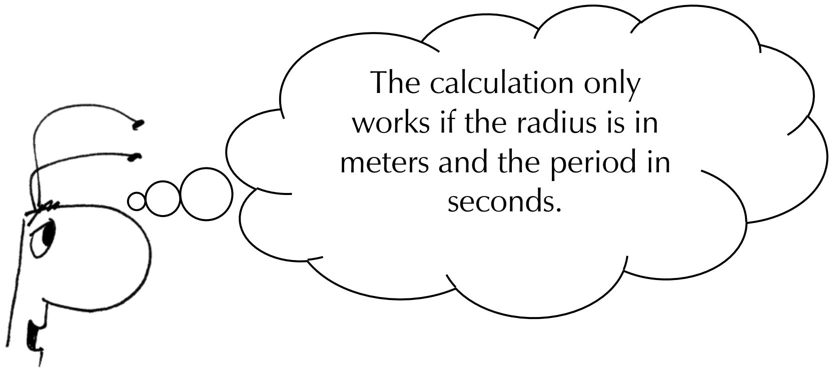
CA1 6.17: The Mass of Jupiter Lab

6. Solve Newton's form of Kepler's Third Law for m. Write the formula here.

$$m = \frac{4\pi^2}{GP^2} r^3$$



7. Finally, plug in the values you got for the radius and period of the moon's orbit into Newton's form of Kepler's third law and determine the mass of Jupiter.

____

____

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____

____

8. Use reference book (in the appendix) values for the orbital period of your moon and its distance from Jupiter to repeat the calculation and write down what you get for the mass of Jupiter here.

____

____

____

9. Look up the mass of Jupiter from the appendix and write it down here.

____

____

____

Print Name ______________________________ Period _________ Date ________________

CA1 6.17: The Mass of Jupiter Lab

10. Determine the relative error of your measurement compared to the textbook measurement using this formula:

____

____

____

____

11. Explain why astronomers doing cutting edge research might not be able to use the relative error formula to determine how well they did their job.

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____

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____

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