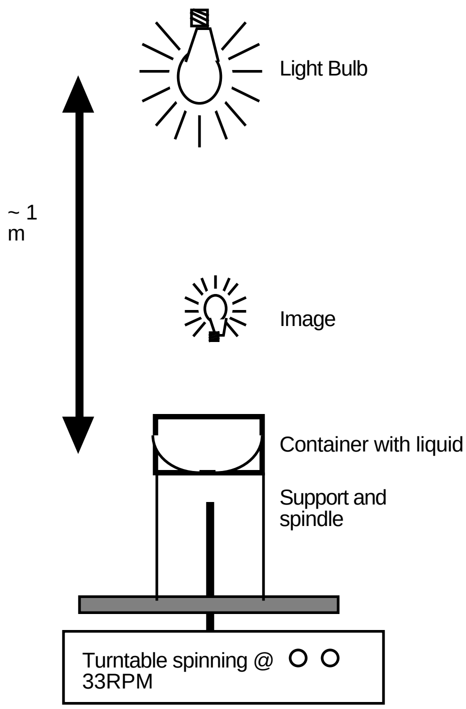
DEMO : Rotating Liquid Mirrors
For AP Physics Institute
by Jeff Adkins

Introduction: Rotating liquid mirrors are used in two astronomical settings. One is a mercury mirror which points at the zenith and is designed primarily to make large aperture, deep sky telescopes at low cost. The other is to spin cast a mirror with molten glass and let the glass cool even as the container spins.

In both cases the molten material assumes a parabolic shape while spinning. The derivation below supports this and the demonstration shows how to present it.



Demo:

Equipment needed: old record player, container of dark Kool-Aid, radially symmetrical container for the liquid, lamp, cardboard, ruler

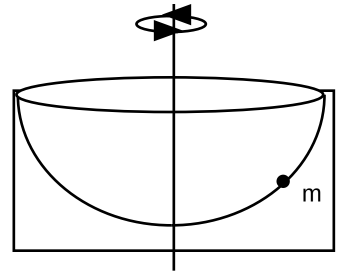
To set up the demo, put a container of reflective liquid (Kool Aid works well, mercury not recommended for high school) centered exactly above the spindle of a record player turntable. You may need to support the container with a box or cylinder which lifts it above the spindle.

In testing, the dark Kool-Aid seemed to cut back on unnecessary reflection from the surface of the white bowl we used. Aluminum paint may also serve before it dries.

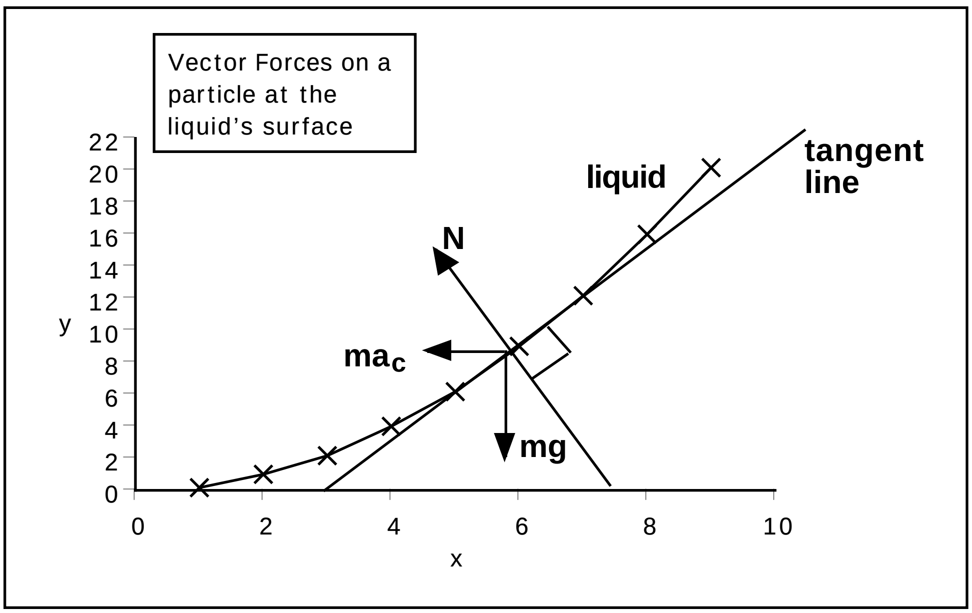
A normal incandescent light bulb suspended about a meter above the liquid will form a real image above the surface which can be shown with an index card or video camera. Marking on the bulb provided a higher-contrast image to focus.

Increasing the angular frequency of the turntable will shorten the focal length of the mirror, which can be easily demonstrated and seen by anyone able to observe the focused image. A darkened room shows the image projected better, and naturally, the larger the mirror, the brighter the image.

Derivation:



Consider a particle on the surface of the rotating liquid as shown. The particle must be in equilibrium if the surface of the liquid is not changing shape. The two forces acting on the particle are its weight, which presses it down on the surface of the liquid, and the centrifugal force experienced by the particle in its rotating reference frame pushing it outward. (We can also complete the derivation with the centripetal force and the reaction to the particle's weight against the water. The result is the same relationship.)



The diagram at left shows the vector components of force on the particle.

From the diagram we can see that the two forces, the normal force from the liquid and the weight mg of the particle combine to form the resultant centripetal force. Since the centripetal force and the weight are perpendicular, we can use them to determine the slope of the Normal line. That is not as useful, however, as the slope of the tangent line, which can be used to infer the shape of the liquid surface using calculus.

Since the tangent line is perpendicular to the normal, we can use the absolute value of the centripetal acceleration as the "rise" and the absolute value of the weight as the "run." Considering only the forces, and cancelling the test mass m that have in common, it can be seen that the slope of the normal vector is- g/a_c and the slope of the tangent line is therefore a_c/g.

1) Therefore, $\dfrac{dx}{dy} = \dfrac{a_c}{g}$

However, a_c is the centripetal acceleration, which is $\omega^2 x$.

Integrating, we obtain the equation of the curved surface:

2) $\int \frac{\omega^2}{g} x dx = \frac{\omega^2 x^2}{2g}$ + a constant, which we can set to zero using the origin at the surface of the liquid.

Comparing this to the equation for a parabola showing the focal length f,

3) $y = \frac{x^2}{4f}$

We can see that the surface meets the criteria of a paraboloid of revolution with focal length defined by the relation

4) $\frac{\omega^2}{2g} = \frac{1}{4f}$

Therefore the focal length is given by

5) $f = \frac{g}{2\omega^2}$

The table below gives the effective focal lengths of the mirrors for 33, 45, and 78 RPM.

Therefore, at 33 RPM, for an object located beyond 41 cm from the rotating mirror, a real image will be formed between 20.5 and 41 centimeters, which easily accomodates the scale of a lecture table. As expected, the faster the turntable spins, the shorter the focal length.

ω	ω	f
(RPM)	(Hz)	(meters)
33	3.455749	0.41
45	4.712385	0.22
78	8.168134	0.07

PRIMARY REFERENCES:

1. Visit to University of Arizona Mirror Lab, 1990; presentation by staff.
2. J. Surdej and A. Pospieszalska-Surdej, "The LMT Didactical Experiment", http://astra.astro.ulg.ac.be/themes/tellins/lmt/didac_e.html, June 20, 2001.