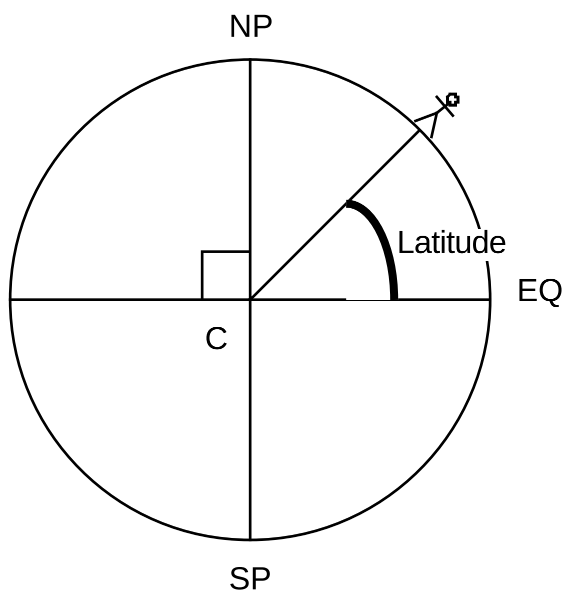
Latitude and the Altitude of Polaris

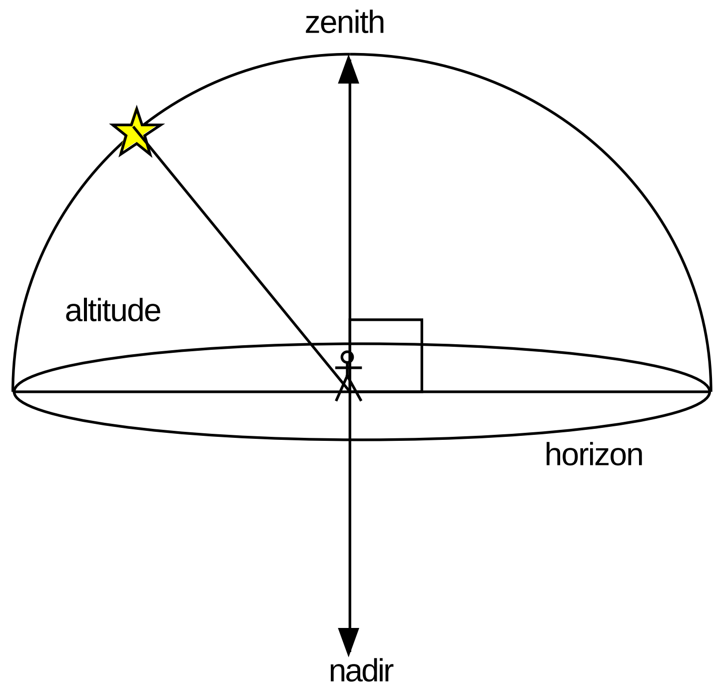
An application of a geometric proof

Navigators have long known that the star Polaris is very close to the North Celestial Pole, that point in the sky which is directly overhead when standing on the North Pole of the Earth. When the observer is on the equator, Polaris is on the horizon; thus is stands to reason that if Polaris is between the horizon and the zenith, the observer must be between the equator and the North Pole. Observing the altitude angle of Polaris is equivalent to measuring one's latitude. This explanation uses geometry to prove this is true (at least, for observers in the Northern Hemisphere.) It also defines the basic astronomy terms used in the paragraph above.



An observer on the earth (Figure 1) is standing on a sphere. Since the earth rotates, is rotation axis is a fixed, convenient place to define positions. Where the axis comes out of the earth we define *poles*, one in the north and one in the south. In the figure, NP is the *North Pole*, and SP is the *South Pole*. The Earth's *equator* is a plane halfway between the poles, passing through the center of the earth and perpendicular to the rotation axis. An observer standing at some mid-northern location has a *latitude* equal to the angle between the observer and the equator using the center of the earth as the vertex.

To the observer, the earth's north pole is not at the "top" of their local view of the world-- *they* are. In other words, to all observers on a spherical planet, the entire planet appears to be underneath their feet. A local observer can define directions not based on the earth's rotation axis, but rather on his or herself. (Figure 2.) Straight up from the observer, as defined by a vertical string from which a weight hangs, is called the *zenith*. Straight down is called the *nadir*, and since the earth is a sphere, it also points to the center of the earth. (The earth is not a perfect sphere, but the extra corrections necessary to compensate for this effect are beyond the scope of this paper.) Similar to the previous definition, the observer's *horizon* is a plane perpendicular to a vertical (zenith-nadir) line, passing through the observer, stretching off to apparent infinity. The *altitude angle of a star*, then, is the angle between the horizon and the star, with the observer as the vertex.



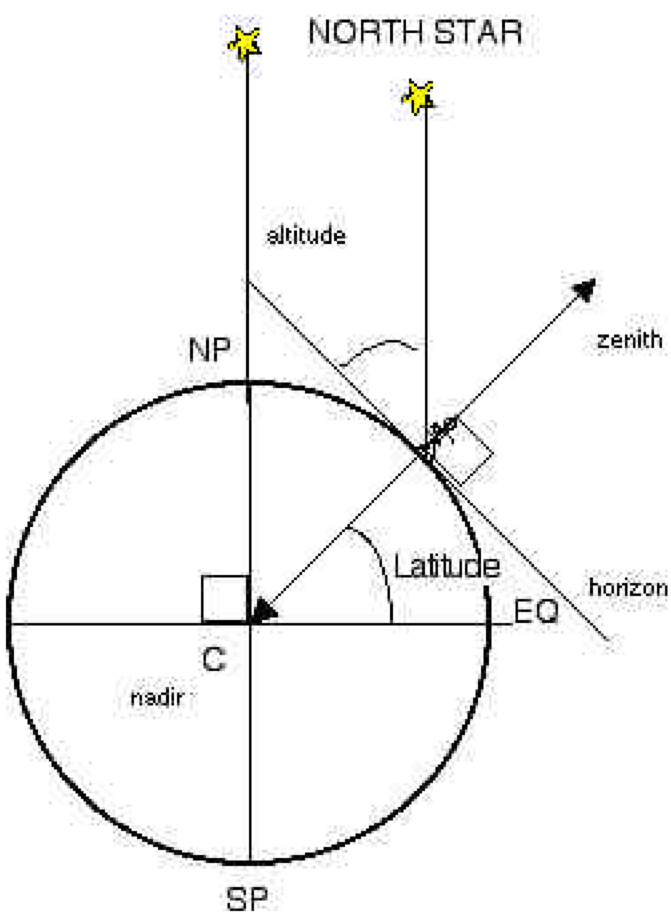


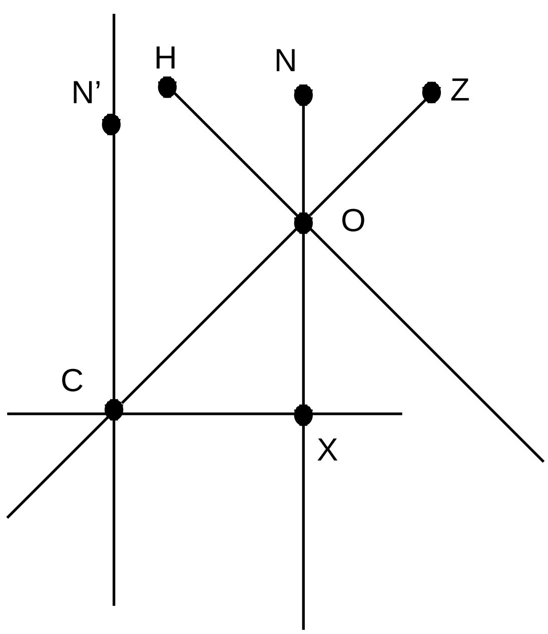
Figure 3 represents acombination of these two diagrams, noting the following conditions are met:

- The observer's nadir points at the center of the earth.
- The observer's horizon is tangent to the circular (spherical) surface of the earth.
- The observer's horizon is perpendicular to the line connecting the observer to the center of the earth because
  a. radius lines from the point of contact of a tangent line are perpendicular to the tangent line and
  b. this is the same as the observer's nadir direction, which is perpendicular to the horizon by definition.

The North Star, Polaris is so far away that it is essentially at an infinite distance; thus, the two observers point in parallel directions when looking at the Pole Star. (This ignores the 1 degree offset between Polaris and the true North Celestial Pole.)

The essential diagram illustrating the situation, with all the extraneous lines removed, is figure 4. Here, the observer (O) has a zenith (in the direction of line segment OZ) and a horizon (line segment (OH) which are perpendicular. The observer is looking at the North Star (N). Point (C) is the center of the earth, and line segment (CN') is the rotation axis of the earth, which points toward the north star. Point X is not illustrated in Figure 3, but occurs when the observer's line of sight to the North Star (ON) is extended into the earth; there is no physical or astronomical significance to this point, but it is necessary to complete the proof.

1) Line segment ON is parallel to line segment CN' because of the distance to Polaris compared to the size of the earth.



2) Line segment CX (the equator) is perpendicular to CN' because the equator is perpendicular to the rotation axis (CN') by definition.
3) OX is also perpendicular to CX, because of 1, 2, and alternate interior angles being equal.
4) Triangle CXO is a right triangle because of the included right angle described in 3).
5) Angles OXC, XCO, and XOC add up to 180 degrees because they are the interior angles of a triangle.
6) Since OXC is a right angle, the remaining two angles (XOC and OCX) must add up to 90 degrees.
7) Angle ZOH is a right angle because of the definitions of horizon and zenith.
8) Angle ZON is equal to angle XOC because they are opposite angles in intersecting lines. (vertical angles)
9) Angle ZOH equals the sum of the angles ZON and NOH; but this is 90, the same as the sum of XOC and OCX.
10)So the sum of ZON and NOH equals XOC and OCX, but ZON is identical to XOC, so we can subtract that from both sides of the equality, leaving...

11) NOH is equal to OCX. NOH i the altitude of Polaris, and OCX is the latitude of the observer.

QED.

I hope purists will forgive my casual use of equality and congruence of segments and angles; in this paper, single letters are points, two letters are segments, and three letters are angles.

Can you figure out a more elegant or a shorter way to express this proof? Send me your ideas.

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